Determining the Relevant Fair Value(s) of S&P 500 Futures

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A fundamental consideration for potential users of stock index futures is the determination of the futures' break-even price or fair value. Conceptually, being able to sell futures at prices above the break-even or buy futures at prices below the break-even offers opportunity for incremental gain. This article points out an important, though widely unappreciated caveat. That is, no single break-even price is universally appropriate. Put another way, the break-even price for a given institution depends on the motivation of that firm as well as its marginal funding and investing yield alternatives.

In this article five differentiated objectives are identified, and the calculations of the respective break-even futures prices are provided. The various objectives are: (a) to generate profits from arbitrage activities, (b) to create synthetic money market instruments, (c) to reduce exposure to equities, (d) to increase equity exposure and (e) to maintain equity exposure using the more cost effective instrument via stock/futures substitution.

All these alternative objectives have the same conceptual starting point, which relates to the fact that a combined long stock/short futures position generates a money market return composed of the dividends on the stock position as well as the basis[1] adjustment of the futures contract. Under the simplified assumptions of zero transaction costs and equal marginal borrowing and lending rates, the underlying spot/futures relationship can be expressed as follows:

(1) $$F = S\left(1 + (i - d)\frac{t}{360}\right)$$

Where F = break-even futures price

S = spot index price

i = interest rate (expressed as a money market yield)

d = projected dividend rate (expressed as a money market yield)

t = number of days from today's spot value date to the value date of the futures contract.

In equilibrium, the actual futures price equals the break-even futures price, and thus the market participant would either have no incentive to undertake the transactions or be indifferent between competing tactics for an equivalent goal.

Moving from the conceptual to the practical simply requires the selection of the appropriate marginal interest rate for the participant in question, as well as precise accounting for transaction costs. This paper demonstrates that these considerations foster differences between the break-even prices among the alternative goals considered. Each goal is explained more fully, and the respective theoretical futures prices are presented.

[1] "Basis" in this paper is defined as the futures price minus the spot index value. Elsewhere, the calculation might be made with the two prices reversed.

Generating Profits from Arbitrage Activities

Generally, arbitrage requires identifying two distinct marketplaces where something is traded, and then waiting for opportunities to buy in one market at one price and sell in the other market at a higher price. This same process is at work for stock/futures arbitrage, but these market participants tend to view their activities with a slightly different slant. They will enter an arbitrage trade whenever (a) buying stock and selling futures generates a return that exceeds financing costs, or (b) selling stock and buying futures results in an effective yield (cost of borrowing) that falls below marginal lending rates. Completed arbitrages will require a reversal of the starting positions, and the costs of both buying and selling stocks and futures must be included in the calculations.[2] Thus, the total costs of an arbitrage trade reflects the bid/ask spreads on all of the stocks involved in the arbitrage, the bid/ask spreads for all futures positions, and all commission charges on both stocks and futures.[3]

Table 1 (page 3) calculates these arbitrage costs under three different scenarios. In all cases, the current starting value of the stock portfolio, based on last-sale prices, is $100 million and the S&P 500 index is valued at 950.00. The size of the hedge ratio is calculated in the traditional manner:[4]

$$H = \frac{V \times Beta}{S\&P \times 250} \qquad (2)$$

Where H = size of the hedge (number of futures contracts required)

V = value of the portfolio

Beta = portfolio beta

S&P = spot S&P 500 index price

250 = the multiplier on the S&P 500 index futures contract, and the average price per share is estimated to be $50.

In column A, transactions are assumed to be costless, reflected by zero values for bid/ask spreads as well as zero commissions. In column B, more typical conditions are shown. Commissions on stock are assumed to be $.02 per share; bid/ask spreads on stocks are assumed to be 1/8 ($.125 per share); commissions on futures are assumed to be $12 on a round-turn basis (i.e., for both buy and sell transactions); and bid/ask spreads on futures are assumed to be two ticks or 0.20, worth $50. Column C assumes the same commission structure as that of column B, but bid/ask spreads are somewhat higher, reflecting a decline in liquidity relative to the former case. This scenario also might be viewed as representing the case where impact costs of trying to execute a stock portfolio were expected to move initial bids or offers for a complete execution. The index point costs in all cases reflect the respective dollar costs on a per-contract basis.[5]

The arbitrageur would evaluate two independent arbitrage bounds: An upper bound and a lower bound. During those times when futures prices exceed the upper arbitrage boundary, profit could be made by financing the purchase of stocks at the marginal borrowing rate and selling futures; and when the futures prices are below the lower bound, profits could be made by selling stocks and buying futures,thus creating a synthetic borrowing, and investing at the marginal lending rate. In both cases, the completed arbitrages would require an unwinding of all the original trades.

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2 If any fees or charges apply to the borrowing or lending mechanisms,these, too, would have to be incorporated in the calculations. Put another way, for the calculations that are presented in this article, the marginal borrowing and lending rates are effective rates,inclusive of all such fees.

3 Brennan & Schwartz (1990) note that the cost of closing an arbitrage position may differ if the action is taken at expiration versus prior to expiration. Thus, the appropriate arbitrage bound should reflect whether or not the arbitrageur is expecting (or hoping) to exercise an “early close-out option.”

4 See Kawaller (1985) for a discussion of the justification for this hedge ratio.

5 In practice, it may be appropriate to assume two different cost structures for the upper- and lower-bound break-even calculations,because costs differ depending on whether the trade starts with long stock/short futures or vice versa. The difference arises because initiating the short stock/long futures arbitrage requires the sale of stock on an uptick. The “cost” of 0this requirement is uncertain because the transactions price is not known at the time the decision is made to enter the arbitrage. No analogous uncertainty exists when initiating the arbitrage in the opposite direction.

TABLE 1: Arbitrage Costs

	A	B	C
S&P 500 Index Value	950.00	950.00	950.00
Size of Portfolio	100,000,000	$100,000,000	$100,000,000
Average Price per Share	50.00	50.00	50.00
Number of Shares	2,000,000	2,000,000	2,000,000
Commission per Share of Stock	0.00	0.02	0.02
Stock Commissions per Side	0.00	40,000	40,000
Stock Commissions (RT)	0.00	80,000	80,000
Bid/Ask Per Unit of Stock	0.00	0.125	0.50
Bid/Ask Stock	0.00	250,000	1,000,000
Contracts	421	421	421
Commissions per Round Turn	0.00	12.00	12.00
Futures Commissions	0.00	5,052	5,052
Bid/Ask per Futures Contract	0.00	0.20	1.00
Bid/Ask Futures	0.00	21,050	105,250
Dollar Costs	0.00	$356,102	$1,190,302
Index Point Cost per Futures Contract	0.00	3.38	11.31
Marginal Borrowing Rate	6.00%	6.00%	6.00%
Marginal Lending Rate	5.00%	5.00%	5.00%
Dividend Rate	3.50%	3.50%	3.50%
Shorter Horizon (Case a):			
Days to Expiration	30	30	30
Upper Bound	951.98	955.36	963.29
Lower Bound	951.19	947.80	939.88
No-Arbitrage Range	0.79	7.56	23.41
Longer Horizon (Case b):			
Days to Expiration	60	60	60
Upper Bound	953.96	957.34	965.27
Lower Bound	952.38	948.99	941.07
No arbitrage range	1.58	8.35	24.20

The upper bound is found by substituting the arbitrage firm's marginal borrowing rate in equation (1) and adding the arbitrage costs (in basis points) to this calculated value. In the case of the lower arbitrage boundary, the marginal lending rate is used for the variable *i* in equation (1), and the arbitrage costs are subtracted. The calculations in Table 1 assume marginal borrowing and lending rates of 6% and 5%, respectively, and a dividend rate of 3.5%. The upper and lower arbitrage boundaries are given for the three alternative cost structures. For comparative purposes, arbitrage boundaries are generated for two different time periods.

Most obvious is the conclusion that an arbitrageur with a higher (lower) cost structure or a wider (narrower) differential between marginal borrowing and lending costs would face wider (narrower) no-arbitrage boundaries. In addition, Table 1 also demonstrates the time-sensitive nature of the difference between the two bounds, or the no-arbitrage range. As time to expiration expands, this range increases monotonically, all other considerations held constant.

Creating Synthetic Money Market Securities

The case of the firm seeking to construct a synthetic money market security by buying stocks and selling futures is a slight variant of the arbitrage case described in the prior section.[6] In this situation, too, the firm will seek to realize a rate of return for the combined long stock/short futures positions, but the relevant interest rate that underlies the determination of the break-even futures price is different. While the arbitrageur who buys stock and sells futures will do so whenever the resulting gain betters the marginal *borrowing* rate, the synthetic fixed-income trader will endeavor to outperform the marginal *lending* rate. For both, however, the imposition of transaction costs will necessitate the sale of the futures at a higher price than would be dictated by the costless case.

Not surprisingly, the break-even price for this player is directly related to both transaction costs and time to expiration. What may not be quite as readily apparent is the fact that, at least theoretically, situations may arise that provide no motivation for arbitrageurs to be sellers of futures, while at the same time offering a motivation for a potentially much larger audience of money managers to be futures sellers. Put another way, large scale implementation of the synthetic money market strategy by many market investors could certainly enhance these participants' returns,but also have the more universally beneficial effect of reducing the range of futures price fluctuation that do not induce relative-price-based trading strategies.

Yet another seemingly perverse condition that is highlighted by these calculations is that firms that operate less aggressively in the cash market, and thereby tend to have lower marginal lending rates, will likely have a greater incremental benefit from arranging synthetic securities than will firms that seek out higher cash market returns. For example, assume Firm A has access to Euro deposit markets while Firm B deals only with lower yielding U.S. domestic banks; and assume further that the difference in marginal lending rates is 0.25%. Firm B's break-even futures price necessarily falls below that of Firm A. At any point in time, however, the current futures bid is relevant for both firms. Assuming the two firms faced the same transaction cost structures, this futures price would generate the same effective yield for the two firms. Invariably, Firm B will find a greater number of yield enhancement opportunities than will Firm A; and any time both firms are attracted to this strategy simultaneously, B's incremental gain will be greater.

Decreasing Equity Exposure

The case of the portfolio manager who owns equities and is looking to eliminate that exposure has two alternative courses of action: He/she could (1) simply sell the stocks, or (2) continue to hold the equities and overlay a short futures position. If the adjustment were expected to be permanent, the first course of action would likely be preferred, as the stocks would have to be liquidated anyway, at some point.Thus, the use of a futures hedge would only delay the inevitable and add additional costs.When the adjustment to the equity exposure is expected to be temporary, on the other hand, the use of futures would likely make more sense, given the significantly lower transactions costs associated with the use of futures versus traditional shares. Even in this case, however, there is a break-even futures price below which the short futures hedge becomes uneconomic, despite the advantageous transaction cost comparison.

This break-even price is found by recognizing that the effect of the hedge is to convert the equity exposure into a money market return.

[6] The case where the firm already holds the stock is considered later.